Math 107

3.2: More on Exponential Growth and Decay

Exponential Formula

- The formula for an exponential function y of t is

$$Y = \text{Initial value} \times \text{Base}^{t}$$

- An exponential function y of t is characterized by the following property: When t increases by 1, to find the new value of y, we multiply the current value by the base.
- A quantity grows exponentially when it increases by a constant percentage over a given period
- If r is the percentage growth per period, expressed as a decimal, the base of the exponential function is 1+r. Note r can be positive (exponential growth) or r can be negative (exponential decay)
- Exponential growth starts slowly and then increases rapidly

Example: You invest $1000 in an account paying 8% per year. Let's work with Excel to see how this effects the investment over time

Example: A population of pine beetles is growing in a matter each week the population doubles Create an Excel Chart and find the formulas for Exponential growth in this case What is the base? What is the percentage growth?

Example: A library reports that journal prices had increased by 150% over a period of 10 years. The library stated that this represented a price increase of 15% each year. Create an Excel Chart and let's see if this statement is correct

Example: The world population in 2010 was about 6.582 billion people. At that time the population was increasing by 1.1% per year. Create a formula to find this change and then use your calculator or Excel to predict the population in 2030?

Practice:

The balance of an investment is increasing according to the rule

Next year's balance = 1.15 x current balance

If the original value of the investment is $500, find a formula that gives the balance after t years

What is the percentage increase each year?

Exponential decay:

Example: The half-life of plutonium-239 is 24,000 years. If you start with 400 grams, how much would be left after 24,000 years? How much would be left after 48,000 years? How much would be left after 96,000 years?

Half Life:
After h half lives, the amount of a radioactive substance remaining is given by the formula

Amount remaining = Initial Amount x $\left(\frac{1}{2}\right)^{h}$

You can find the amount remaining after t years by first expressing t in terms of half lives and then using the formula above

Example: the isotope known as Carbon-14 is radioactive will decay into the stable form nitrogen-14. Assume that percentage of carbon-14 in the air over the past 50,000 years remained constant. As long as an organism is alive, it ingests air and the level of carbon-14 remains the same. When it dies, it no longer absorbs carbon-14 form the air, and the carbon-14 in the organism decays, with a half-life of 5770 years.

Suppose a prehistoric tree contained c grams of carbon 14 when it was cut down. What percentage of the original amount of carbon-14 would be find if it was cut down 30000 years ago?